*Reference: F&H chapter 18*

Introduction:

- Over half the known nuclei have configurations **(Z,N) even, $J^\pi = 0^+$**
- Recall that an empirical pairing term is included the semi-empirical mass formula to account for their unusual stability.

N.B. The pairing term is **not** accounted for in the shell model, which ignores all interactions between particles!!!

- It costs too much energy to break a pair of nucleons and populate higher single particle states, so the excitations of even-even nuclei tend to be of a **collective nature**

→ the nuclear matter distribution as a whole exhibits quantized vibrations in some cases and rotations in others, with characteristic frequency patterns.

- Vibrational spectra are seen in nuclei that have an intrinsic spherical shape
- Rotational excitations tend to occur in nuclei with permanent quadrupole deformations

Vibrational states: 

**Model: quantized oscillations of a liquid droplet** at constant density
(why? repulsive short-distance behaviour of the N-N force!)

Consider oscillations about a spherical equilibrium shape, with a time-dependent boundary surface expressed as a linear combination of spherical harmonic functions:

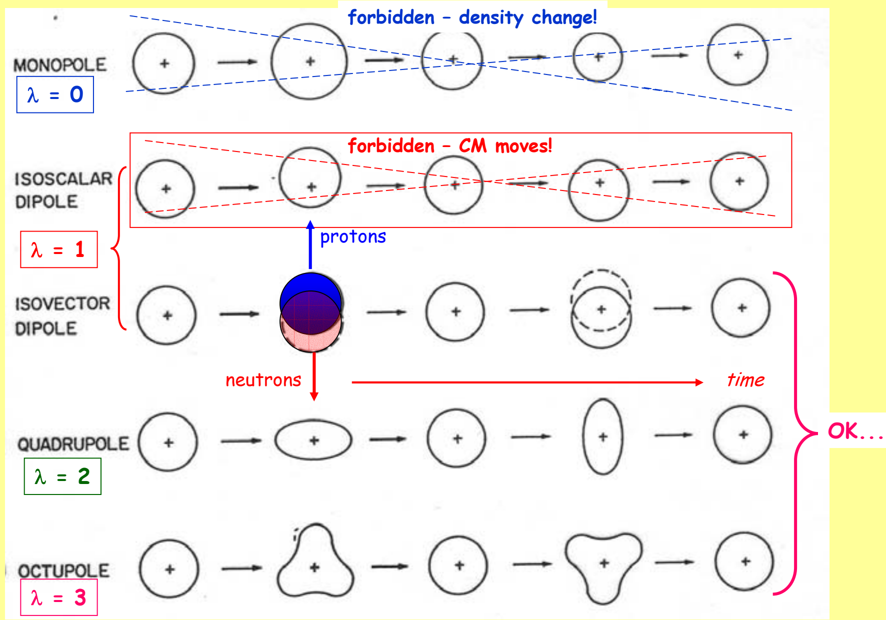
$$R(\theta,\phi,t) = R_o \left\{ 1 + \sum_{\lambda\mu} \alpha_{\lambda\mu}(t)\, Y_{\lambda\mu}(\theta,\phi) \right\}$$

expansion describes any shape at all, given appropriate coefficients. Each contribution can in principle oscillate at a different frequency....

Normal modes of the system correspond to excitations with a particular value of $\lambda$ and $\mu$, and these will occur at characteristic frequencies.

Application to nuclei:
1. restriction to axial symmetry, i.e. $\mu = 0$
2. vibrations are quantized: $E_n = n\,\hbar\omega$

Illustration: time sequence of oscillating nuclear shapes 3




"Giant Dipole Resonance" in Nuclei: $J^\pi = 1^-$ "GDR" 4

- characteristic feature that can be seen in all nuclei
- very short-lived state at high excitation
- $E_1 = \hbar\omega_1 \cong 78\ A^{-1/3}$ MeV (example below: 197Au, E = 15 MeV)
- $\Gamma \cong 6$ MeV (common feature) → $\tau \cong 10^{-22}$ s
- classical analog is an oscillating electric dipole moment – can decay via E1 radiation pattern (signature)

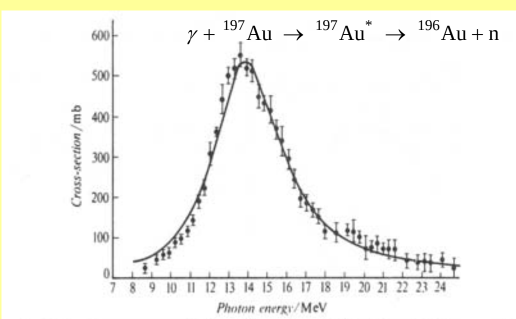



Fig. 8.10 Giant resonance of photodisintegration in $^{197}$Au. The yield of neutrons is shown as a function of the energy of the monochromatic photons used to produce the reaction (Fultz, S. C. *et al.*, *Phys. Rev.*, **127**, 1273, 1963).

Electric dipole radiation pattern:

*Griffiths, Intro. to Electodynamics:*

z $\langle \vec{S} \rangle \sim \frac{\sin^2\theta}{r^2}\hat{r}$

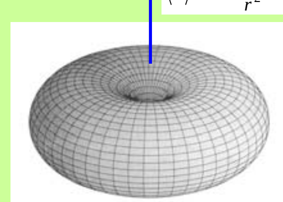
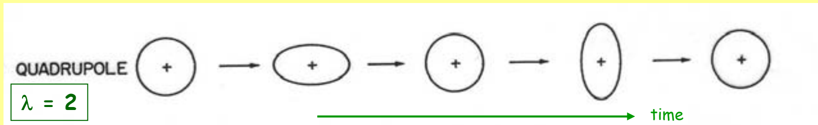


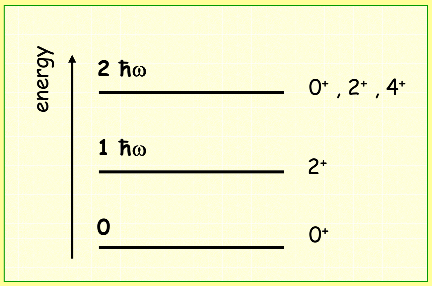
Figure 11.4

## Quadrupole oscillations occur at lower energy: $J^{\pi} = 2^{+}$

5




- typically, $\hbar\omega_2 \sim 1$ MeV in a variety of even-even nuclei
- excitation energy is low, so can expect to see up to several "quadrupole phonons" in the spectrum
- Boson excitations, so require a symmetric wave function under exchange of "particle" (phonon) labels → this restricts the total $J^{\pi}$,




e.g. for two phonons:

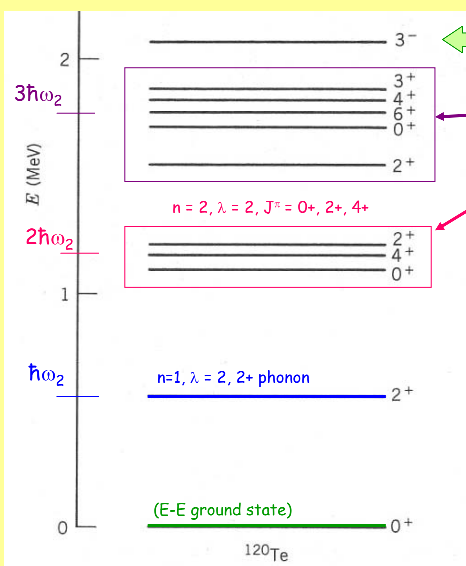
$$\left(\vec{2} + \vec{2}\right)_{symmetric} = \vec{J}$$

$$\Rightarrow \quad J = 0,\ 2,\ 4\ \ only$$

Model spectrum

## Example of vibrational excitations: $^{120}_{52}Te_{68}$

6

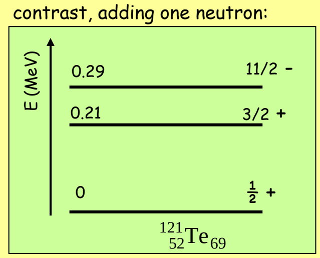



3- state?

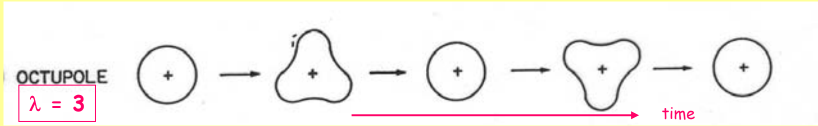
multiple λ = 2 phonon states, ideally degenerate

(characteristic decay patterns: gamma-rays emitted with "E2" angular distributions, like a quadrupole antenna)

contrast, adding one neutron:



The $3^-$ state is an octupole phonon, $\lambda$ = 3: 7



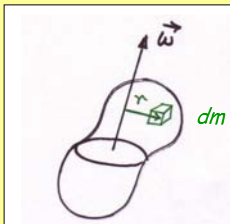

- $J^\pi = 3^-$
- $\hbar\omega_3 \sim$ (2-3) $\hbar\omega_2 \sim$ 2 – 3 MeV
- typically only see one octupole phonon per spectrum

Summary:

- low lying excitations in even-even spherical nuclei have the same characteristic pattern up to a few MeV in excitation energy:

0+ (gs), 2+ (quadrupole phonon), (0+, 2+, 4+) (two phonons), 3- (octupole)

## 2. Quantized Rotations in deformed nuclei: 8




Classical rotating object:

$$E = \frac{L^2}{2I}$$

$$I = \int r^2 \, dm$$

Replace L with rotational angular momentum J:

moment of inertia sets the scale of the energy level pattern

$$|J^2| = J(J+1) \Rightarrow E_J = \frac{\hbar^2}{2I} J(J+1)$$

allowed J determine a characteristic spacing pattern

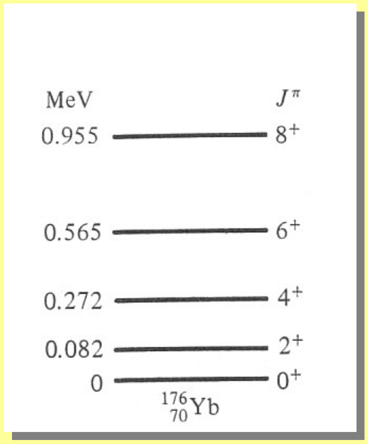
J is quantized; "rotational bands" are spectra characterized by a given value of the moment of inertia, I, and a series of energy levels with $\Delta J$ = 1 or 2:

- even-even nucleus: J = (0, 2, 4, 6, 8, 10....) $\pi$ = +
- odd-even deformed nucleus: J = ½ integer, $\Delta J$ = 1 within a "band"

## Example: $^{176}$Yb

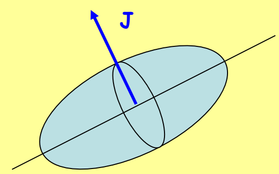


Quantized energy states of a rotating football!




$$E_J = \frac{\hbar^2}{2\,I}\,J(J+1)$$

$$\Rightarrow \frac{\hbar^2}{2\,I} \cong 0.014 \text{ MeV}$$

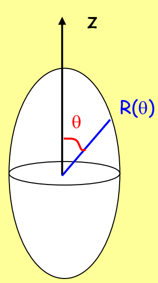



Note -- larger I means smaller energy level spacing!

note: rotations around the symmetry axis are indistinguishable; rotational angular momentum must be perpendicular to the symmetry axis.

## The moment of inertia gives a measure of the nuclear shape:






parameterize the shape, quadrupole moment and moment of inertia assuming constant density football shape:

$$R(\theta) = R_o\,(1+\beta\,Y_{20}(\theta))$$

$$\beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\,\frac{\Delta R}{R_o} \cong 1.05\,\frac{\Delta R}{R_o}$$

$$Q_{zz} = \frac{3}{\sqrt{5\pi}}\,R_o^2\,Z\beta\,(1+0.16\beta)$$

If the nucleus rotates like a solid: *(rigid body model):*

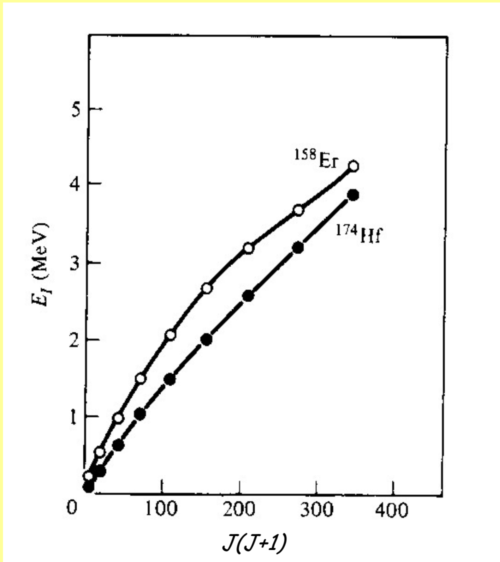
$$\boxed{I_R = \frac{2}{5}\,M\,R_o^2\,(1+0.31\beta)}$$

If the nucleus rotates like a liquid drop: *(rotating fluid model)*

$$\boxed{I_F = \frac{9}{8\pi}\,M\,R_o^2\,\beta}$$

reality is somewhere in between...

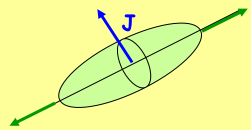
## Evidence of "quasi-fluid" behaviour for rotating nuclei: 11




spectral analysis:

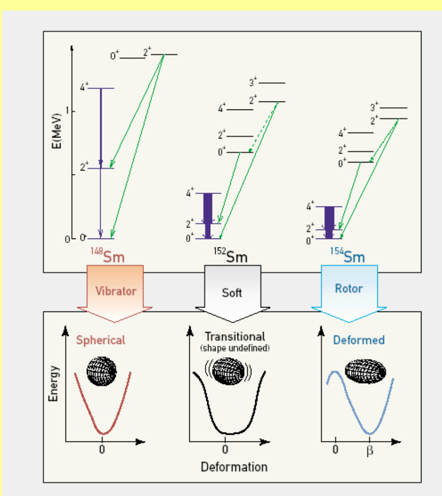
a plot of E vs J(J+1) should give a straight line with slope $\hbar^2/2I$

- confirmed for $^{174}Hf$
- but for $^{158}Er$, the slope decreases (moment of inertia increases) with increasing J ...




Like a rotating fluid: "centrifugal stretching" along the symmetry axis occurs for larger angular momentum!

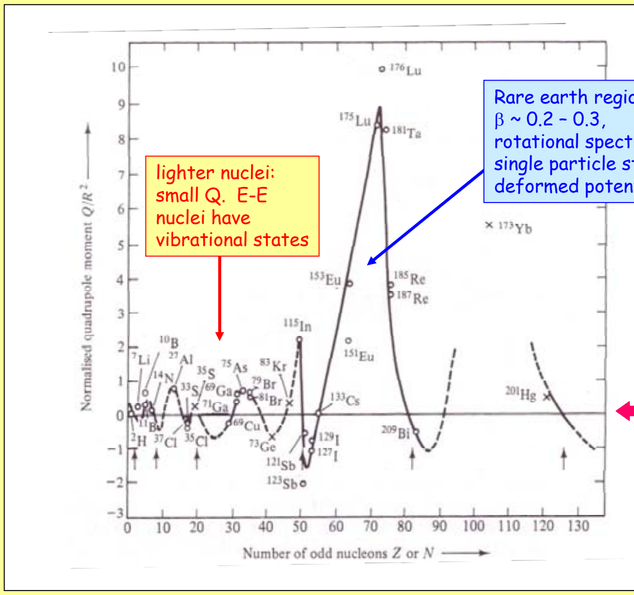
## Current Nuclear Structure studies: Excerpt from US Long Range Plan, p. 35 12




**Shapes of samarium.** The energy-level schemes for three excited isotopes of samarium are shown at the top of the figure, and the inferred nuclear shapes are shown below. The isotope $^{148}Sm$ shows features characteristic of a spherical vibrator, whereas $^{154}Sm$ exhibits rotational bands typical of a deformed (elongated) nucleus. On the other hand, $^{152}Sm$ behaves like a critical-point system, whose shape cannot be precisely defined. This is illustrated in the energy diagrams below the shapes, where energy is plotted as a function of shape deformation. Well-defined minima exist for two of the isotopes, but the energy minimum for $^{152}Sm$ is very broad, and it is impossible to say whether the nucleus is spherical or deformed.

web link: see lecture 1

## Quadrupole moments and types of excitations across the nuclear chart: 13

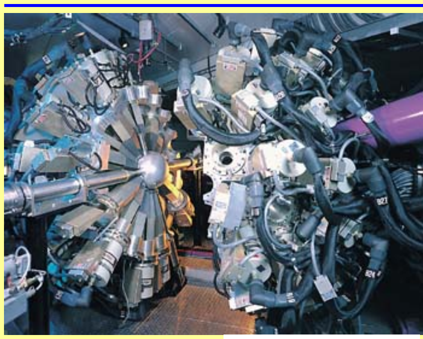



lighter nuclei: small Q. E-E nuclei have vibrational states

Rare earth region:
$\beta \sim 0.2 - 0.3$,
rotational spectra for E-E
single particle states in a deformed potential for O-E

Closed shell nuclei: spherical; O-E nuclei near closed shells are good "Shell Model" candidates...

## High resolution Gamma Detection 14



Gammasphere Detector

e.g. "8 $\pi$" detector array at TRIUMF,
"EUROBALL" array in France, "Gammasphere" UK

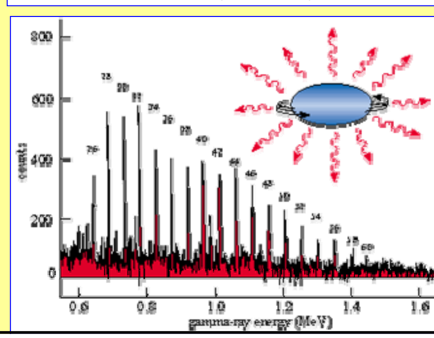
• high resolution detectors: Ge crystals

• surrounded by low resolution, high efficiency scintillation detectors (typically NaI) to reject events where only part of the energy is deposited in the Ge crystal due to Compton scattering

• full angular coverage for both → "8 $\pi$ detection!

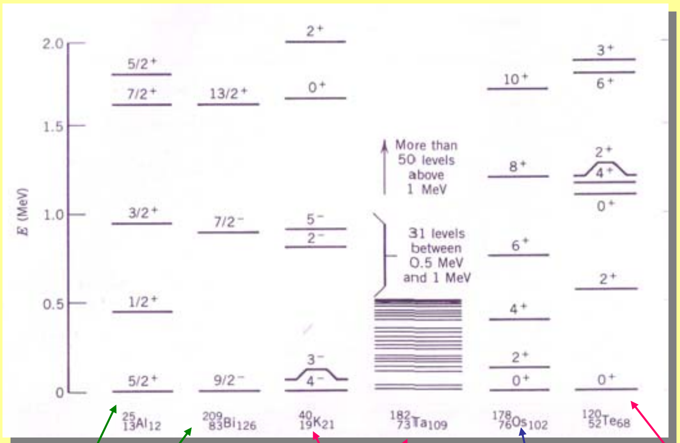
Great current interest:
"Superdeformed Nuclei" (2:1 axis ratio)

Physics World, July 1998: Superdeformed nuclei of dysprosium-152 decay by emitting a regular spectrum of gamma-rays. The number above each transition is the angular momentum quantum number, which decreases by two each time a photon is emitted. The photon carries $h$/p of angular momentum away from the nucleus, which slows down the rotation. After emitting approximately 20 such gamma-rays the nucleus abruptly loses its deformation.
Data from CLRC Daresbury Laboratory, UK



Example – nuclear states:

15



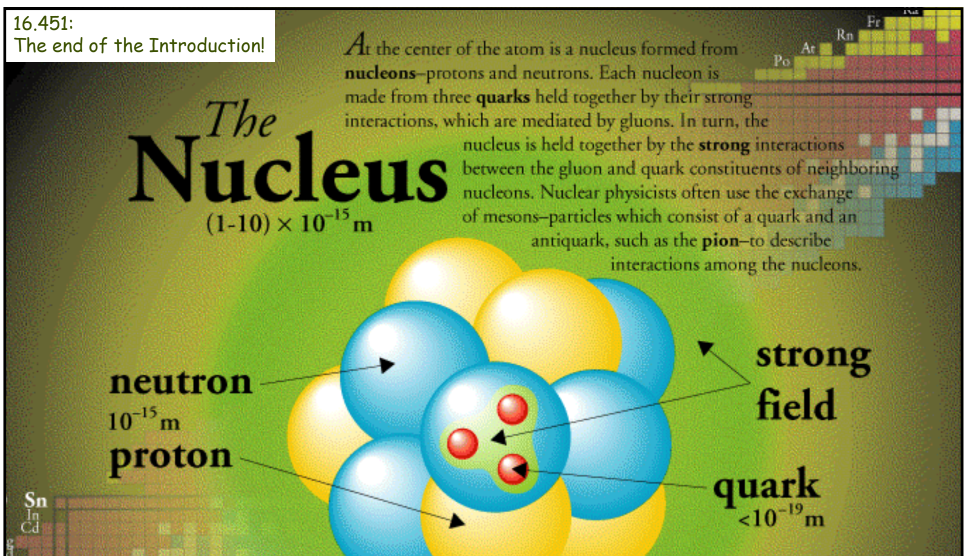

light nuclei, 1 valence nucleon - good shell model description

odd-odd – more than one valence nucleon -- complicated!!!

rotational spectrum

vibrational spectrum

16.451:
The end of the Introduction!



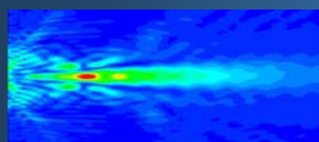

From the U.S. Long range plan : "Opportunities in Nuclear Science" (lecture 1):
The nucleus is a remarkable quantal system displaying diverse and rare phenomena. Governed by the strong interactions among nucleons, nuclei exhibit correlations resulting in both single-particle and collective modes of excitation. Nuclear structure theory attempts to understand these excitations and the responses of nuclei to different external probes, within a coherent framework. This theoretical framework must encompass a wide range of energy and momentum scales for nuclei ranging from the deuteron to the superheavy elements. Theory strives to describe the structure and dynamics of these often-disparate systems and to apply the understanding thus achieved to unravel some of the mysteries of the universe....

## Graduate Studies & Research

- Astronomy & Astrophysics
- Atomic & Molecular Physics
- Condensed Matter & Materials Physics
- Mass Spectrometry of Biomolecules
- Medical Physics
- Physics of Nanoscale Systems
- Subatomic Physics

*CURRENT PROJECTS IN SUBATOMIC PHYSICS:*
*theory of nucleon structure;*
*symmetry tests of strong & weak interactions;*
*nuclear astrophysics,*
*precise atomic mass determinations;*
*ion & atom trapping ....*

*STRATEGIC LINKS:*
*TRIUMF, Argonne & Los Alamos National Labs,*
*Jefferson Lab, Max Planck Inst. ....*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